THE DISSIPATIVE CRITERION OF CREEP RUPTURE

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Most promising and extensively utilized to describe the process of metal creep at high temperatures is, at this time, the hypothesis of the equations of state of Rabotnov [1]. According to this hypothesis, the creep strain rate tensor $\dot{p}_{ij}$ is determined by the running values of the stress tensor σ_{ij}, the temperature T, and the structural parameters $q_1, q_2, \ldots, q_n$, whose changes can be described by the appropriate kinetic equations

$$\dot{p}_{ij} = \dot{p}_{ij}(\sigma_{kl}, T, q_1, q_2, \ldots, q_n), \ \dot{q}_m = \dot{q}_m(\sigma_{kl}, T, q_1, q_2, \ldots, q_n).$$

Making these relationships and the structural parameters specific permits describing all three stages of the creep process, or some of them.

Let us henceforth limit ourselves to the one-dimensional case at a constant temperature so as not to investigate the question of the influence of the kind of stress state or the variability of the temperature on the creep process.

To describe the creep rupture process in [1-4], one scalar structural parameter $\omega(t)$ was used. In substance, this "microstructural" parameter reflects the degree of internal damage of the material and varies between zero in the undamaged state and one at the time of rupture. Selected as such a parameter in [5-7] is the magnitude of the energy $D(t) = \int \sigma\dot{p}dt$ dissipated during creep, which is the "macrostructural" parameter reflecting the dissipative nature of the creep process and taking its limit value at the time of rupture.

1. Let us examine the creep process as a unit and interaction between two irreversible cumulative processes, the cumulative creep strain p(t) and the cumulative damage $\omega(t)$. From the thermodynamic viewpoint, these two processes are elements of one irreversible energy scattering process. As in [5-7], we assume that this energy takes on its limit value at rupture. The concept of the "effective" stress $\sigma/(1 - \omega)$ [2, 8] is ordinarily introduced for materials that become brittle during creep. The concept of the scattering power for a unit "effective" volume can also be introduced, which is related to the "effective" stress $\sigma/(1 - \omega)$ by the relationship $\dot{D}(t) = \sigma\dot{p}(1 - \omega)$. Then the kinetic equations for p(t), $\omega(t)$, D(t) and the rupture criterion are written in the form

$$\dot{p} = B\left(\frac{\sigma}{1-\omega}\right)^n, \quad p(0) = 0; \tag{1.1}$$

$$\dot{\omega} = A\left(\frac{\sigma}{1-\omega}\right)^m, \quad \omega(0) = 0; \tag{1.2}$$

$$\dot{D} = B\left(\frac{\sigma}{1-\omega}\right)^{n+1}, \quad D(0) = 0; \tag{1.3}$$

$$\int_0^{t_*} \dot{D}dt = D_* = \text{const}. \tag{1.4}$$

Here A, B, m, n, D_* are temperature-dependent material parameters. We integrate (1.1)-(1.3) under constant σ. We obtain

$$p(t) = \frac{B}{A}\sigma^{n-m}\frac{1-(1-\omega(t))^{m+1-n}}{m+1-n}; \tag{1.5}$$

$$\omega(t) = 1 - \left(1 - A(m+1)\sigma^m t\right)^{\frac{1}{m+1}}; \tag{1.6}$$

$$D(t) = \frac{B}{A}\sigma^{n+1-m}\frac{1-(1-\omega(t))^{m-n}}{m-n}. \tag{1.7}$$

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TABLE 1

σ, MPa	40	50	60	80
t_*, h	54/54,1	23,5/26,7	15,5/14,8	6/5,7
p_*, %	12,6/13,0	10,0/11,5	8,2/10,2	12,4/8,3

TABLE 2

σ, MPa	100	115	130	150	180
t_*, h	444/348	211/211	141/136	65/81	38/42
p_*, %	71/69	51/65	69/62	60/58	47/52

Taking account of the rupture criterion (1.4), we have for the values of the time to rupture $t_*(\sigma)$ and the strain at the time of rupture $p_*(\sigma)$ from (1.5) and (1.6):

$$t_* = \frac{1-(1-\omega_*(\sigma))^{m+1}}{A(m+1)\sigma^m}; \tag{1.8}$$

$$p_* = \frac{B}{A}\sigma^{n-m}\frac{1-(1-\omega_*(\sigma))^{m+1-n}}{m+1-n}, \tag{1.9}$$

where $\omega_*(\sigma)$ is the magnitude of the damage at the time of rupture (expressed in terms of D_*):

$$\omega_*(\sigma) = 1-\left(1-(m-n)\frac{AD_*}{B}\sigma^{m-n-1}\right)^{1/(m-n)}. \tag{1.10}$$

The boundedness of $\omega_*(0<\omega_*<1)$ and p_* for any σ imposes the following constraint $m \leqslant n$ on m and n. This condition is less rigorous than the condition $m \leqslant n < m+1$ that results from the boundedness of p_* for the rupture criterion $\omega_*=1$ [1].

The dependence $t_*(\sigma)$ in (1.8) is depicted in the logarithmic coordinates lg σ — lg t_* (the creep strength curve) as a convex curve with two rectilinear asymptotes corresponding to the dependences $t_*=1/(A(m+1)\sigma^m)$ (brittle fracture), and $t_*=D_*/B\sigma^{n+1}$ (viscous fracture). The point of intersection of these lines $\sigma_0=[A(m+1)D_*/B]^{1/(n+1-m)}$ is a certain stress having conditional boundaries between the viscous and brittle fractures for creep. The dependence $p_*(\sigma)$ in (1.9) has a growing section $p_* \sim \sigma^{n-m}$ for small σ and a decreasing section $p_* \sim \sigma^{-1}$ for large σ, i.e., qualitatively reflects the experimental dependence $p_*(\sigma)$ in a sufficiently large range of stresses [9]. It is seen from (1.9) that σp_* grows as does σ, and approaches its limit value D_*. This effect (the diminution in the scattered energy at the time of rupture as t_* grows, or equivalently, as σ diminishes) was observed in a number of experiments [7].

2. Since the values of m are sufficiently large, as a rule, the quantity $(1-\omega_*)^{m+1}$ in (1.8) can be neglected as compared with one. This simplification permits a sufficiently simple determination of the parameters A, B, m, n, and D_*. The dependences $\dot{p}_{min}(\sigma)$, $p_*(\sigma)$, and $t_*(\sigma)$ are determined experimentally in a series of uniaxial creep tests up to rupture. The parameters B and n are determined by least squares from the dependence $\dot{p}_{min}(\sigma)$. Analogously (taking account of the assumption on the smallness of $(1-\omega_*)^{m+1}$ as compared to one), the parameters A and m are found from the dependence $t_*(\sigma)$. Let us express D_* in terms of p_* and σ in (1.9) and (1.10)

$$D_* = \frac{B}{A}\sigma^{n+1-m}\frac{1-\left(1-(m+1-n)\frac{Ap_*}{B}\sigma^{m-n}\right)^{\frac{m-n}{m+1-n}}}{m-n}. \tag{2.1}$$

TABLE 3

σ, MPa	62	93	124	139	155	186	217
t_*, h·10^3	415/421	139/141	61,2/65,2	46/47,5	34,6/35,8	21,2/21,9	14,1/14,5
p_*, %	9,0/8,9	11,0/10,9	12,2/12,6	13,4/13,4	14,1/14,0	15,5/15,3	16,8/16,5

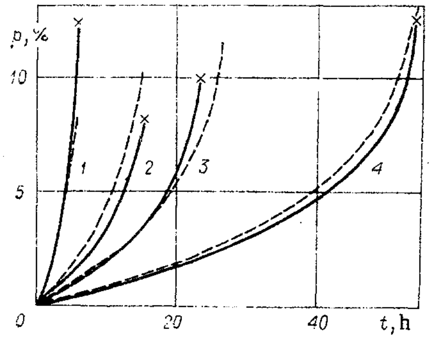

Fig. 1

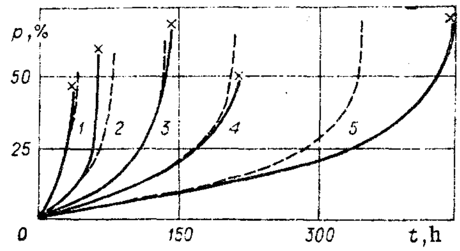

Fig. 2

The relationship (2.1) permits independent processing of the dependences $p_*(\sigma)$ and determination of the last parameter D_* as the mean in the whole range of stress variation.

Let us examine a number of materials for which the experimental dependences $\dot{p}_{min}(\sigma)$, $p_*(\sigma)$, $t_*(\sigma)$, and the creep curves are presented.

1. Steel Kh18N10T at 850°C and σ =40–80 MPa ([3]). For it m =3.07, n =3.2, $A(m+1)$ = $0.22\cdot10^{-6}$ $(\text{MPa})^{-m}$ h^{-1}, B +$0.63\cdot10^{-8}$ $(\text{MPa})^{-n}$ h^{-1}, D_* =8,5 MPa, σ_0 =170 MPa.

2. Titanium alloy OT-4 at 500°C and σ =100–180 MPa ([7], data taken from [4]). For it $m=n$ =3.59, $A(m+1)$ =$1.9\cdot10^{-10}$ $(\text{MPa})^{-m}$ h^{-1}, B =$0.33\cdot10^{-10}$ $(\text{MPa})^{-n}$ h^{-1}, D_* =154 MPa, σ_0 =890 MPa.

3. Low-alloy steel at 500°C and σ =62–217 MPa ([10]). For it m = 2.69, n =3.2, $A(m+1)$ = $0.36\cdot10^{-10}$ $(\text{MPa})^{-m}$ h^{-1}, B =$0.52\cdot10^{-13}$ $(\text{MPa})^{-n}$ h^{-1}, D_* =$2.1\cdot10^{3}$ MPa, σ_0 =$12\cdot10^{3}$ MPa.

Experimental values of t_* and p_* (numerator) and those determined from (1.8) and (1.9) (denominator) are compared in Tables 1-3 for these materials, respectively. The dashed lines in Figs. 1 and 2 display the theoretical creep curves (1.5), while the solid lines are the experiment data. Curves 1-4 in Fig. 1 (for steel Kh18N10T at 850°C) correspond to σ =80, 60, 50, 40 MPa. Curves 1-5 in Fig. 2 (for the alloy OT-4 at 500°C) correspond to σ =180, 150, 130, 115, 100 MPa.

It is seen from the data presented that the first two materials are in the mixed rupture domain, while the third is in the brittle fracture domain. Therefore, the deduction can be made that the dissipative creep rupture criterion is applicable for metals that tend to brittleness. The creep strength curve is here approximated by two rectilinear lines in the lg σ — lg t_* coordinates, and the possibility is manifest for independent processing of the experimental data for $p_*(\sigma)$ and an indication of the brittle, mixed, and viscous fracture domains.

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